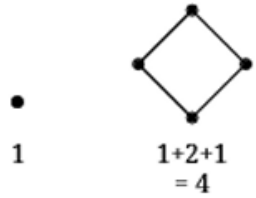
1.4. Relations among Number Sequences

Figure it out

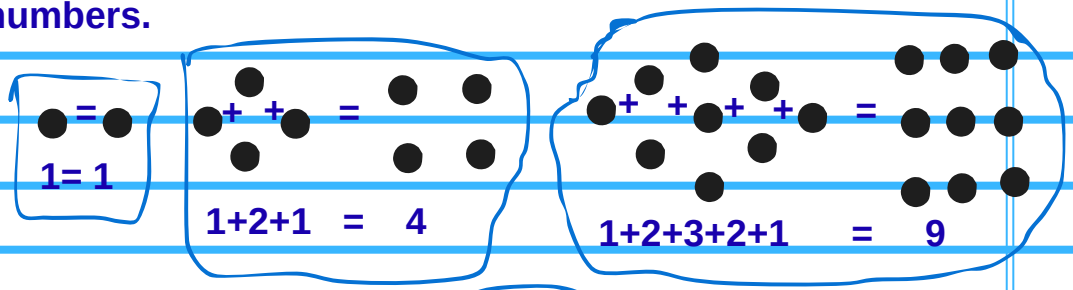
Pg. 8 Ex. 1.4.1

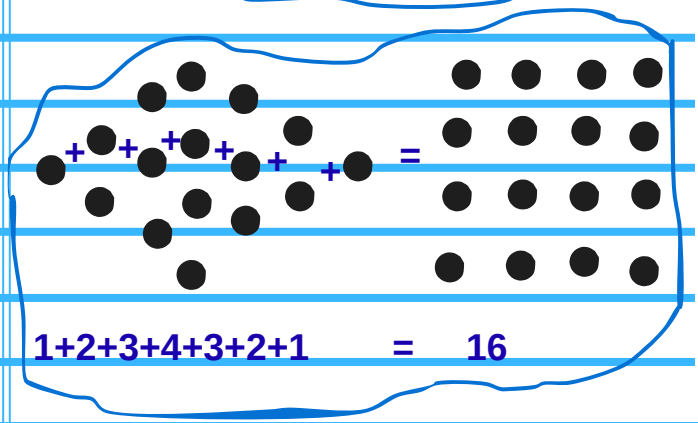
Can you find a similar pictorial explanation for why adding counting numbers up and down, i.e., 1, 1 + 2 + 1, 1 + 2 + 3 + 2 + 1, …, gives square numbers?

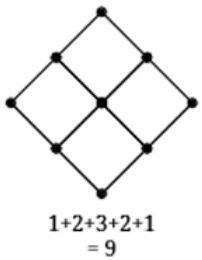


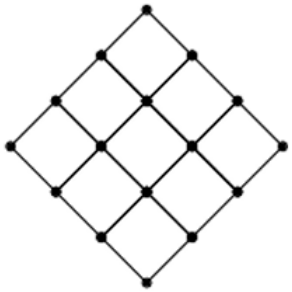
Soln:

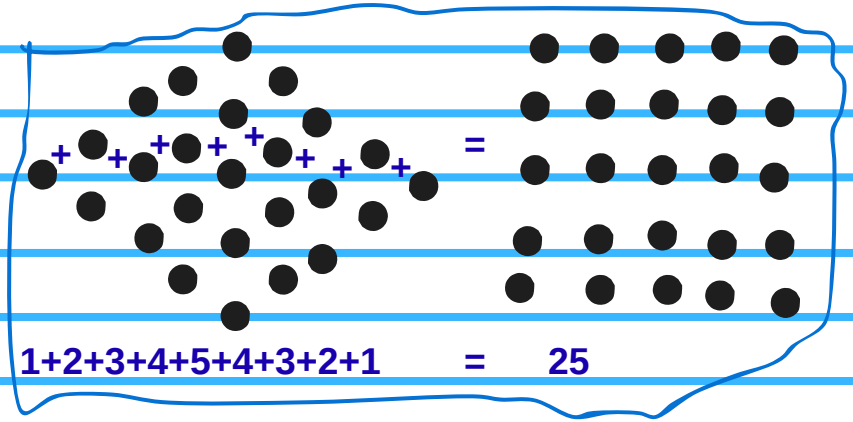
Yes, I can find a similar pictorial explanation for adding counting numbers up and down, gives square numbers.











Pg. 8 Ex. 1.4.2 (Diagram is attached in next page. Kindly refer and draw)

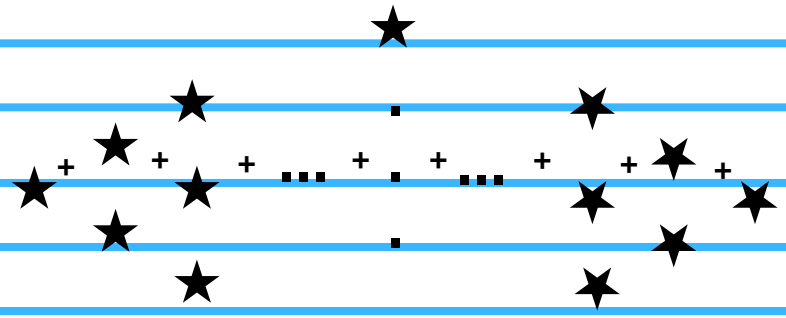
By imagining a large version of your picture, or drawing it partially, as needed, can you see what will be the value of 1 + 2 + 3 + ... + 99 + 100 + 99 + ... + 3 + 2 + 1?

(Must draw diagrams in exam, as it is asked in question)

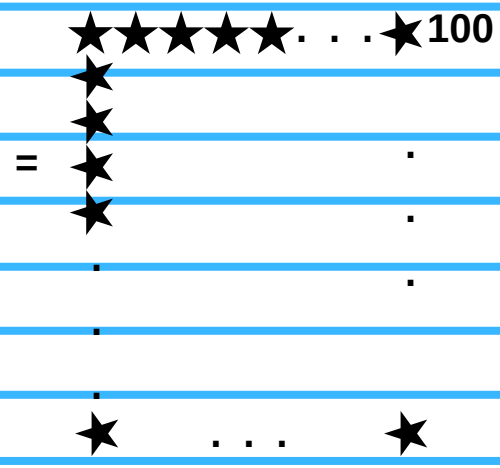
Soln:

We know that, as we see in the above 1.4.1 adding couting numbers up and down will give square numbers. So, 1+2+3+...+99+100+99+...+3+2+1 will be 10000. Because 100^2 = 100 x 100 = 10000

Refer the diagrams in 1.4.1, we clearly see that, adding the counting numbers, let us take counting number five, add it up and down, we get its square which is 25. Similarly, we said for 100 its 10000.



1 + 2 + 3 + ... + 100 + ... + 3 + 2 + 1



10000

Pg. 8 Ex. 1.4.3

Which sequence do you get when you start to add the All 1's sequence up? What sequence do you get when you add the All 1's sequence up and down?

(Need not to draw diagram, as it didn't asked in question, however if asked in more scorer, you can draw else can draw for good presentation)

Soln:

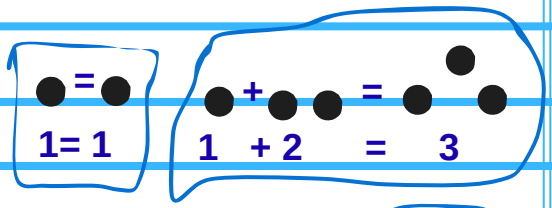
I get Counting Numbers sequence when I start to add the all 1's sequence up.	I get Odd Numbers sequence when I add the all 1's sequence up and down.
1=1	1=1
1+1=2	1+1+1=3
1+1+1=3	1+1+1+1+1=5
1+1+1+1=4	1+1+1+1+1+1+1=7
1+1+1+1+1=5	1+1+1+1+1+1+1+1+1=9
.	.
.	.
.	.

Pg. 8 Ex. 1.4.4

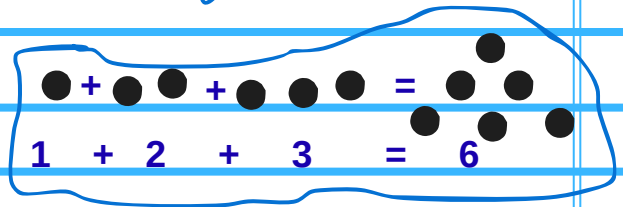
Which sequence do you get when you start to add the counting numbers up? Can you give a smaller pictorial explanation?

(Must draw diagrams in exam, as it is asked in question)

Soln:



I get Triangular Numbers sequence when I start to add the Counting Numbers up.



1=1

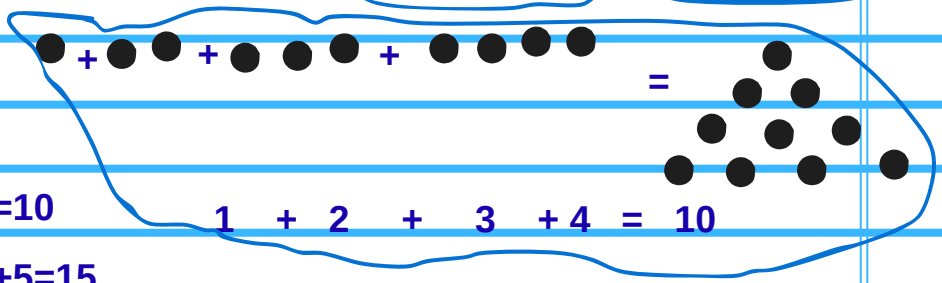
1+2=3

1+2+3=6

1+2+3+4=10

1+2+3+4+5=15

.
.
.



Pg. 8 Ex. 1.4.5

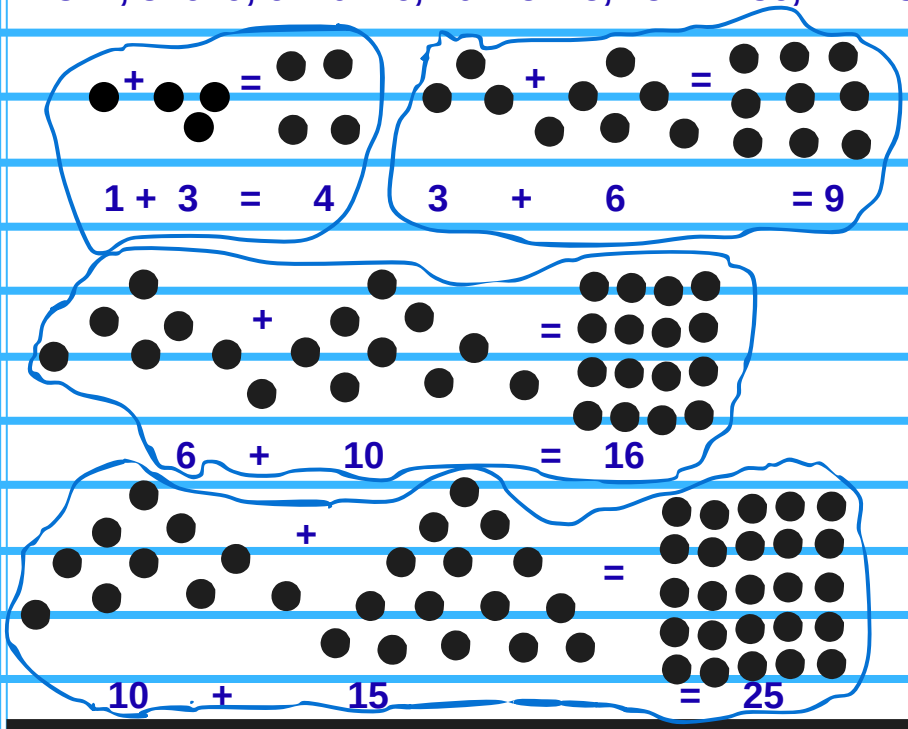
What happens when you add up pairs of consecutive triangular numbers? That is, take 1 + 3, 3 + 6, 6 + 10, 10 + 15, … Which sequence do you get? Why? Can you explain it with a picture?

(Must draw diagrams in exam, as it is asked in question)

Soln:

I get Square Numbers sequence when I add up pairs of consecutive Triangular Numbers.

1+3=4, 3+6=9, 6+10=16, 10+15=25, 15+21=36, 21+28=49,...



Pg. 8 Ex. 1.4.6

What happens when you start to add up powers of 2 starting with 1, i.e., take 1, 1 + 2, 1 + 2 + 4, 1 + 2 + 4 + 8, … ? Now add 1 to each of these numbers — what numbers do you get? Why does this happen?

(Need not to draw diagram, as it didn't asked in question, however if asked in more scorer, you can draw else can draw for good presentation)

Soln:

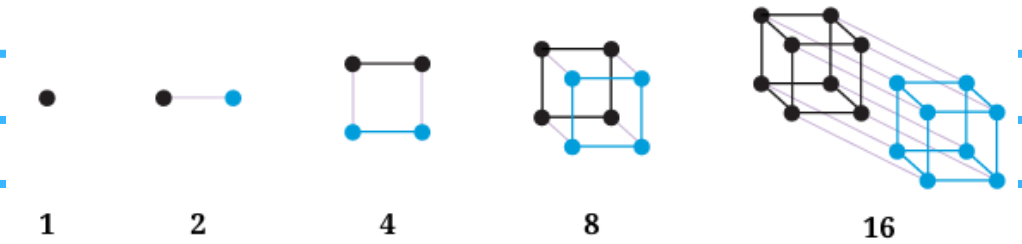
I get the following sequence when I add up powers of 2 starting with 1,

1,3,7,15,31,63,... (subtracting 1 from the numbers in the sequence of powers of 2 starting from 2)

(i.e. 1, 1+2=3, 1+2+4=7, 1+2+4+8=15, 1+2+4+8+16=31, 1+2+4+8+16+32=63,...)

After adding 1 to each number, we get:2, 4, 8, 16, 32,...

(i.e. 1+1=2, 3+1=4, 7+1=8, 15+1=16, 31+1=32,...) (i.e. we get the powers of 2. Refer the diagram in Pg.6 Ex.1.3.5.)



Pg. 8 Ex. 1.4.7

What happens when you multiply the triangular numbers by 6 and add 1? Which sequence do you get? Can you explain it with a picture?

(Must draw diagrams in exam, as it is asked in question)

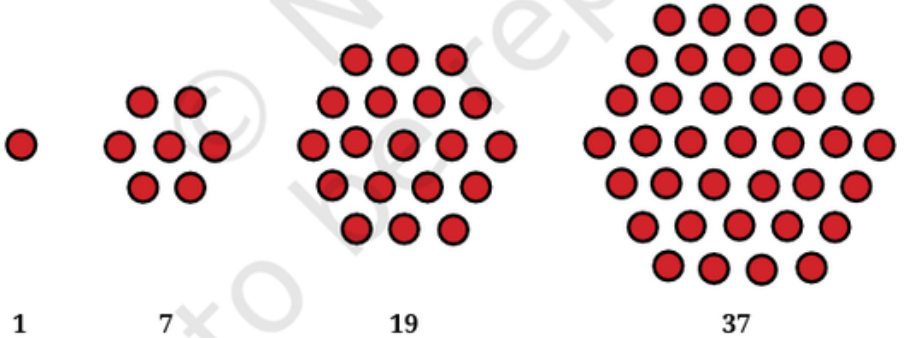
Soln:

I get the sequence of Hexagonal Numbers when I multiply the triangular numbers by 6 and add 1,

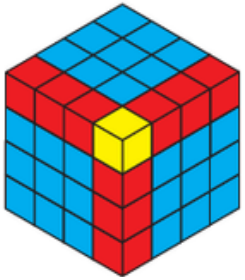
7, 19, 37, 61, 91, 127,…

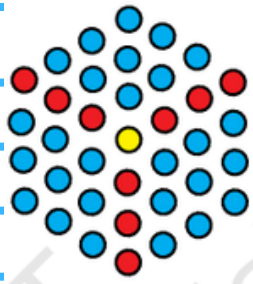
(i.e. (1×6) +1, (3×6) +1, (6×6) +1, (10×6) +1, (15×6) +1, (21x6) +1, …)

Refer the diagram in Pg.5 Ex.1.3.4.



Pg. 8 Ex. 1.4.8





What happens when you start to add up hexagonal numbers, i.e., take 1, 1 + 7, 1 + 7 + 19, 1 + 7 + 19 + 37, … ? Which sequence do you get? Can you explain it using a picture of a cube?

(Must draw diagrams in exam, as it is asked in question)

Soln:

When I start to add up Hexagonal numbers, i get the sequence of cubes.

1

1+7=8

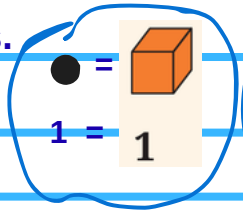
1+7+19=27

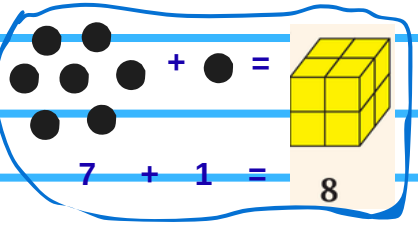
1+7+19+37=64

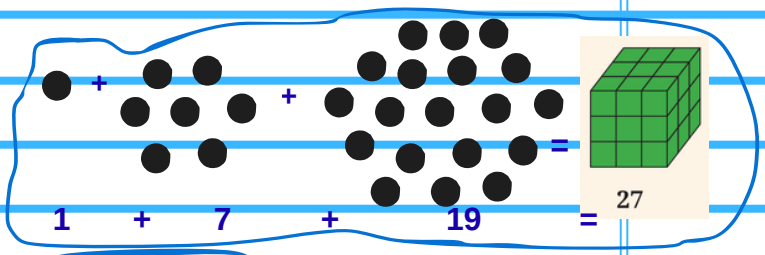
1+7+19+37+61=125

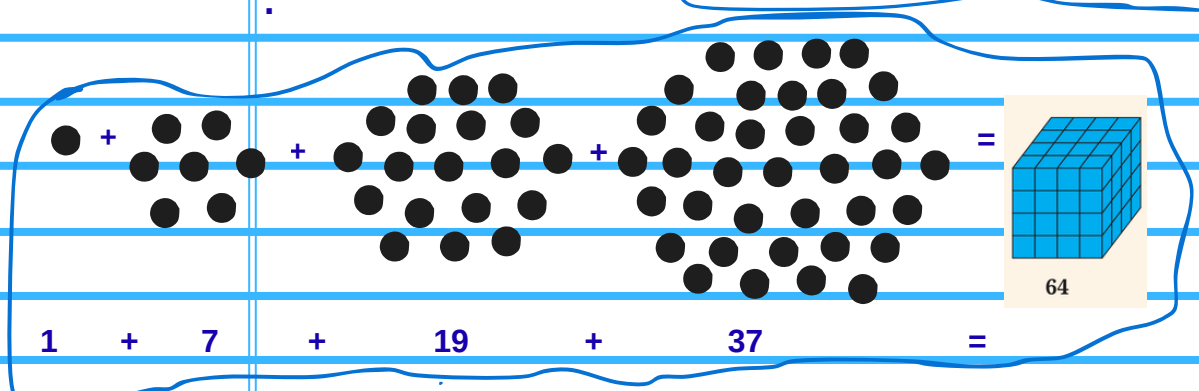
1+7+19+37+61+91=216

⋮









Pg. 8 Ex. 1.4.9

Find your own patterns or relations in and among the sequences in Table 1. Can you explain why they happen with a picture or otherwise?

(Must draw diagrams in exam, as it is asked in question)

Soln:

When I start to add up sequence of all 2's, i get the sequence of even numbers.

2=2

2+2=4

2+2+2=6

2+2+2+2=8

2+2+2+2+2=10

2+2+2+2+2+2=12

2+2+2+2+2+2+2=14

⋮

When I start to add up and down sequence of all 2's, i get the sequence of alternative even numbers.

2=2

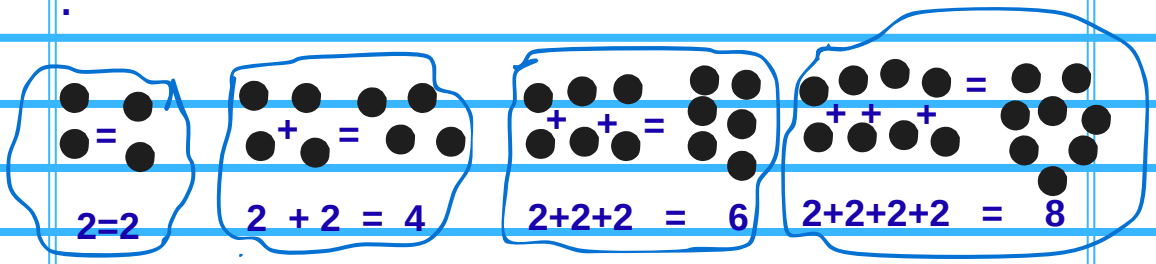
2+2+2=6

2+2+2+2+2=10

2+2+2+2+2+2+2=14

2+2+2+2+2+2+2+2+2=18

⋮



Pg. 11 Ex. 1.5.1

1.5. Patterns in Shapes

Figure it out

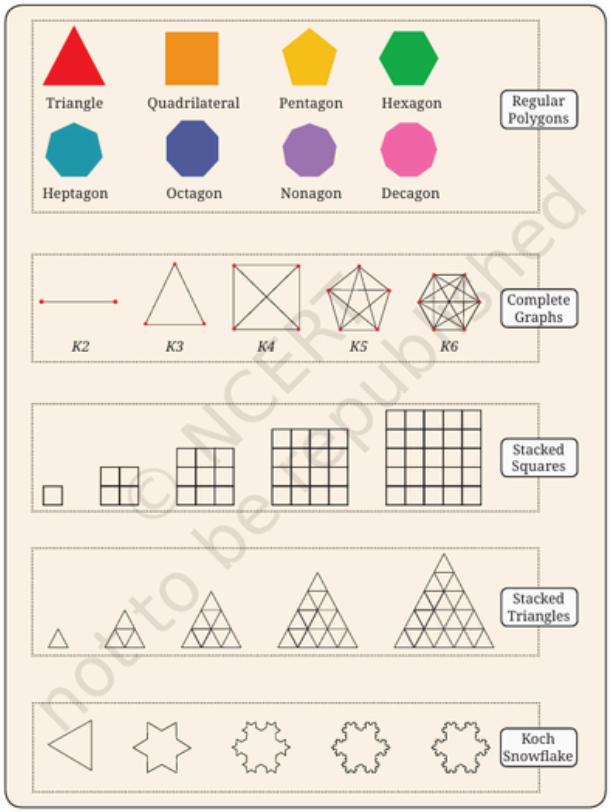
Can you recognise the pattern in each of the sequences in Table 3?

(Must draw diagrams in exam, as it is asked in question)

Soln:

Yes, I can recognise the pattern in each of the sequences in Table 3.

Table 3: Examples of shape sequences



3,4,5,6,7,8,9,10
(sides of the picture)

1,3,6,10,15,...
(Triangular numbers)

1,4,9,16,25,...
(Squares)

1,4,9,16,25,...

3, 3x4, 3x4x4, 3x4x4x4, 3x4x4x4x4,...
(Koch Snowflakes)

Pg. 11 Ex. 1.5.2

1.5. Patterns in Shapes

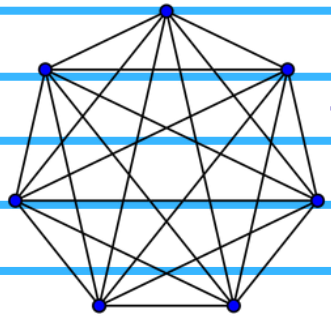
Try and redraw each sequence in Table 3 in your notebook. Can you draw the next shape in each sequence? Why or why not? After each sequence, describe in your own words what is the rule or pattern for forming the shapes in the sequence

(Must draw diagrams in exam, as it is asked in question)

Soln:

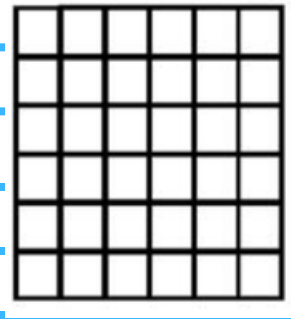
Refer the diagram drawn in Ex. 1.5.1. Here are the next shape in each sequence and the rule of pattern.

3,4,5,6,7,8,9,10,11,12,13,14,...
The sequence represents the sides of the picture or polygon.



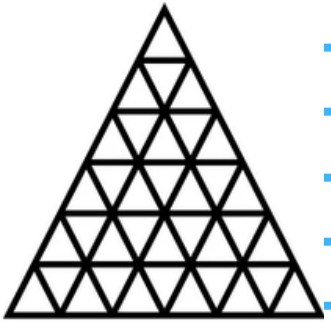
1,3,6,10,15,21,28,3645,55,66...
The sequence is Triangular numbers. It will give us the number of edges on complete Graph. (number of lines present here)

K7 Complete Graph



1,4,9,16,25,36,49,64,81,100,...
The sequence is Square numbers. The representation of square numbers are given in stacked squares.

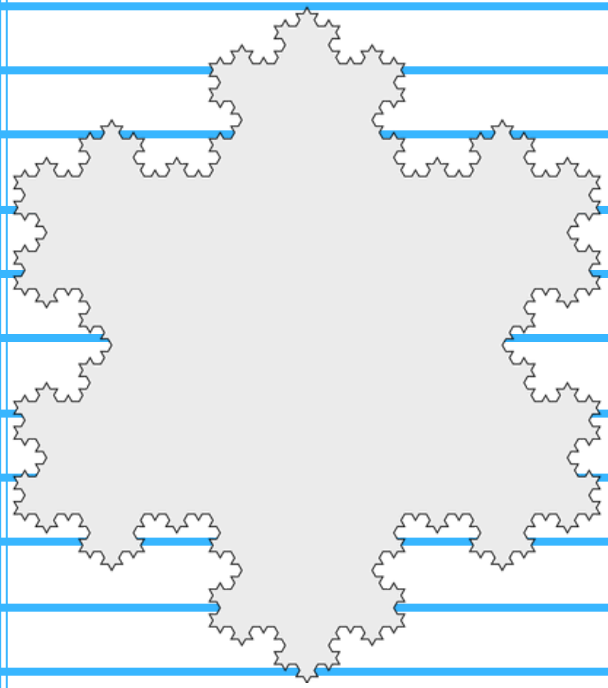
Stacked Squares (36)



1,4,9,16,25,36,49,64,81,100,...
The sequence is Square numbers. The representation of square numbers are given in stacked triangles.

Stacked Triangles (36)

(continued in next page)



Koch snowflake
3x4x4x4x4x4

The sequence is Koch snowflake 3, 3x4, 3x4x4, 3x4x4x4, 3x4x4x4x4, 3x4x4x4x4x4, 3x4x4x4x4x4x4,...

To get from one shape to the next shape in the Koch Snowflake sequence, one replaces each line segment '—' by a 'speed bump' __/__. As one does this more and more times, the changes become tinier and tinier with very very small line segments.

1.6. Relation to Number sequences

Figure it out

Pg. 11 Ex. 1.6.1

Count the number of sides in each shape in the sequence of Regular Polygons. Which number sequence do you get? What about the number of corners in each shape in the sequence of Regular Polygons? Do you get the same number sequence? Can you explain why this happens?

(Need not to draw diagram, as it didn't asked in question, however if asked in more scorer, you can draw else can draw for good presentation)

Soln:

While counting the number of sides in each shape in the sequence of Regular Polygons, I get a sequence of Counting numbers starting from 3.

Number of sides = 3,4,5,6,7,8,9,10…

The number of corners in each shape in the sequence of Regular Polygons is same as the number of sides i.e., 3,4,5,6,7,8,9,10,...

Number of corners = 3,4,5,6,7,8,9,10…

in any closed figure, number of sides = number of corners (vertices).

1.6. Relation to Number sequences

Figure it out

Pg. 11 Ex. 1.6.2

Count the number of lines in each shape in the sequence of Complete Graphs. Which number sequence do you get? Can you explain why?

Soln:

While counting the number of lines in each shape in the sequence of Complete Graphs, I get a sequence of Triangular numbers.

Number of lines = 1, 3, 6, 10, 15…

In simple words, Complete Graphs is a graph or a picture of polygon (shapes) with all of its vertices (corners) connected to each other.

(Need not to draw diagram, as it didn't asked in question, however if asked in more scorer, you can draw else can draw for good presentation)

1.6. Relation to Number sequences

Figure it out

Pg. 12 Ex. 1.6.3

How many little squares are there in each shape of the sequence of Stacked Squares? Which number sequence does this give? Can you explain why?

Soln:

1, 4, 9, 16, 25 number of little squares are there in each shape of the sequence of Stacked Squares. This gives us a Square number sequence. Squares can be drawn using these number of dots. (we already did this earlier in the previous exercises like 1.3.1 and table 2) . We can represent square numbers in the shape of square.

,1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ...

(Need not to draw diagram, as it didn't asked in question, however if asked in more scorer, you can draw else can draw for good presentation)

1.6. Relation to Number sequences

Figure it out

Pg. 12 Ex. 1.6.4

How many little triangles are there in each shape of the sequence of Stacked Triangles? Which number sequence does this give? Can you explain why? (Hint: In each shape in the sequence, how many triangles are there in each row?)

(Need not to draw diagram, as it didn't asked in question, however if asked in more scorer, you can draw else can draw for good presentation)

Soln:

1, 4, 9, 16, 25 number of little triangles are there in each shape of the sequence of Stacked Triangles. This gives us a Square number sequence as adding the counting numbers up and down. i.e., 1, 1+2+1, 1+2+3+2+1,…. (we can see this in the previous exercises like 1.4.1 and on page 7). We can represent square numbers in the shape of square.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ...

1.6. Relation to Number sequences

Figure it out

Pg. 12 Ex. 1.6.5

To get from one shape to the next shape in the Koch Snowflake sequence, one replaces each line segment '—' by a 'speed bump' . As one does this more and more times, the changes become tinier and tinier with very very small line segments. How many total line segments are there in each shape of the Koch Snowflake? What is the corresponding number sequence? (The answer is 3, 12, 48, ..., i.e., 3 times Powers of 4; this sequence is not shown in Table 1.)

(Need not to draw diagram, as it didn't asked in question, however if asked in more scorer, you can draw else can draw for good presentation)

Soln:

Total line segments in each shape is: 3, 12, 48, 192, 768

Corresponding sequence is: 3, 3×4, 3×4×4, 3×4×4×4×, 3×4×4×4×4,…

We can see this in 1.5.2.